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A POWER SPECTRUM PROGRAM FOR ESTIMATING THE DOPPLER PROFILE OF A RADIO CHANNEL

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ABSTRACT

This research memorandum describes a data processing technique for analyzing the Doppler-shift properties of a radio channel. A computer program has been developed that computes the power spectrum of power scattered by the radio channel as a function of Doppler shift. This program was developed in conjunction with phase-stable CW experiments which were performed on two HF paths. The experiments were part of a simulation experiment conducted by Stanford Research Institute for the United States Army Electronics Laboratories and the Defense Atomic Support Agency.[1]*

The program computes the power spectrum of the time variations of the response of the radio channel to a CW tone relative to the transmitted frequency.

*References are given at the end of the memorandum.

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ILLUSTRATIONS

I INTRODUCTION

In a study recently conducted at Stanford Research Institute, the natural radio channel is viewed as a randomly time-varying linear filter.[2] The purpose of this study was to develop mathematical channel models that would enable one to analyze the performance of various communication systems in the presence of signal perturbations induced by the propagation medium.

In this study the radio channel is represented by a time-variant channel transfer function, $H(t,f)$, which is a complex random quantity that is wide-sense stationary in both time and frequency. This function is simply defined as follows. If the input to the channel is

$$x(t) = \mathrm{Re}\left\{e^{i2\pi(f + f_0)t}\right\} \tag{1a}$$

then the output is

$$z(t) = \mathrm{Re}\left\{H(t,f)e^{i2\pi(f + f_0)t}\right\} \quad . \tag{1b}$$

Complex envelope notation is employed in Eq. (1); the symbol Re $\{\ \}$ denotes the real part of the complex quantity in the braces, and f_0 is some reference frequency.

The second-order statistical variations of $H(t,f)$ are summarized by the time-frequency autocorrelation function:

$$R_H(\alpha,\beta) = E\left[H^*(t,f)\, H(t + \alpha,f + \beta)\right] \tag{2}$$

where the symbol $E\left[\;\right]$ denotes the mathematical expectation of the quantity in the brackets. The Fourier transform of $R_H(\alpha,\beta)$ is called the channel scattering function:

$$S(\lambda,\tau) = \iint e^{-i2\pi(\lambda\alpha - \tau\beta)} R_H(\alpha,\beta)\, d\alpha d\beta \quad . \tag{3}$$

The channel scattering function is the density of power scattered by the channel to the Doppler frequency λ and the time delay τ.[2] Thus the channel scattering function yields an immediate physical characterization of the frequency- and time-dispersive properties of the propagation medium. The function

$$T(\tau) = \int S(\lambda,\tau)\, d\lambda \tag{4}$$

is called the channel delay profile, since it yields the density of power scattered by the channel as a function of time delay. Similarly, the function

$$D(\lambda) = \int S(\lambda,\tau)\, d\tau \tag{5}$$

is called the channel Doppler profile, since it yields the density of power scattered by the channel as a function of Doppler shift.

II THE CHANNEL DOPPLER PROFILE

The channel Doppler profile is the power spectrum of the wide-sense stationary time variations of the channel transfer function. From Eqs. (3) and (5) one can deduce that the Doppler profile can be written as the Fourier transform of the autocorrelation function $R_H(\alpha,0)$. Observe that this autocorrelation function ignores the wide-sense stationary frequency variations of the channel transfer function.

$$E\left[H^*(t,f)\ H(t+\alpha,f)\right] = R_H(\alpha,0) \qquad (6)$$

$$D(\lambda) = \int R_H(\alpha,0)\ e^{-i2\pi\lambda\alpha}\ d\alpha \quad . \qquad (7)$$

Hence the Doppler profile is merely the power spectrum of the time variations of the channel response (amplitude and phase) to a CW tone at a fixed frequency. It is evident that the Doppler profile can be estimated with the amplitude and phase measurements obtained from a phase-stable CW experiment, since such an experiment measures $H(t,f)$.

It is convenient to introduce the quadrature components of $H(t,f)$:

$$H(t,f) = U(t) + iV(t) \quad . \qquad (8)$$

In Eq. (8) the frequency dependence of the quadrature components is suppressed because the frequency f will remain fixed in all arguments that follow. The autocorrelation function $R_H(\alpha,0)$ can be expressed in terms of the autocorrelation functions associated with the quadrature components:

$$R_H(\alpha,0) = R_{UU}(\alpha) + R_{VV}(\alpha) + i\left\{R_{UV}(\alpha) - R_{VU}(\alpha)\right\} \tag{9}$$

where

$$R_{UU}(\alpha) = E\left[U(t)U(t+\alpha)\right]$$

$$R_{UV}(\alpha) = E\left[U(t)V(t+\alpha)\right]$$

and similarly for $R_{VV}(\alpha)$ and $R_{VU}(\alpha)$. In most cases of practical interest,[2*]

$$R_{UU}(\alpha) = R_{VV}(\alpha)$$

$$R_{UV}(\alpha) = -R_{VU}(\alpha)$$

which implies that the autocorrelation function $R_H(\alpha,0)$ can be written:

$$R_H(\alpha,0) = 2R_{UU}(\alpha) + i2R_{UV}(\alpha) \quad . \tag{10}$$

By using Eq. (7) and the fact that $R_{UU}(\alpha)$ is an even function of α and $R_{UV}(\alpha)$ is an odd function of α, the Doppler profile is obtained in the form

$$D(\lambda) = 2\int R_{UU}(\alpha)\cos 2\pi\lambda\alpha d\alpha + 2\int R_{UV}(\alpha)\sin 2\pi\lambda\alpha d\alpha \quad . \tag{11}$$

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*These conditions are necessary for wide-sense stationarity of the process $\mathrm{Re}\left\{H(t,f)e^{i2\pi(f+f_0)t}\right\}$ in time.

Observe that the Doppler profile is symmetrical about zero if and only if $R_{UV}(\alpha)$ is zero for all α. When

$$x(t) = \mathrm{Re}\left\{e^{i2\pi(f + f_0)t}\right\}$$

is the input to the channel, the (two-sided) spectrum $S_z(u)$ of the resulting narrow-band output process

$$z(t) = \mathrm{Re}\left\{H(t,f)e^{i2\pi(f + f_0)t}\right\}$$

can be expressed in terms of the Doppler profile

$$S_z(u) = \frac{1}{4}\, D(u - \{f_0 + f\}) + \frac{1}{4}\, D(- u - \{f_0 + f\}) \quad . \qquad (12)$$

Thus, if $D(\lambda) = 0$ for $|\lambda| > f + f_0$, the Doppler profile is merely a frequency-shifted version of the narrow-band spectrum $S_z(u)$.

III DESCRIPTION OF THE POWER SPECTRUM PROGRAM

The input to the power spectrum program that computes the channel Doppler profile consists of two data sets; each data set corresponds to N equispaced samples of one of the quadrature components. The corresponding sample mean is subtracted from each member in each data set, and the result is then prewhitened. Prewhitening of sample data improves the results of the spectral averaging which is performed later in the program.[3] Let W(t) be the result of prewhitening U(t); then the prewhitening characteristic used in the program relates W(t) and U(t) in the following simple manner:

$$W(t_i) = U(t_i) - 0.6U(t_{i-1}) \quad . \tag{13}$$

This prewhitening characteristic multiplies the input spectrum by the quantity[3]

$$1.36 - 1.20 \cos 2\pi T\lambda$$

where T is the sampling period. Observe that the high frequencies are emphasized relative to the low frequencies.

The next step in the power spectrum program computes the sample autocovariances[4,5] and crosscovariances

$$\bar{R}_{UU}(\tau_i) \quad , \quad \bar{R}_{UV}(\tau_i) \quad , \quad \bar{R}_{VV}(\tau_i) \quad , \quad \text{and} \quad \bar{R}_{VU}(\tau_i)$$

where

$$\bar{R}_{UU}(\tau_i) = \frac{1}{N} \sum_{j=1}^{N-i} U(t_j)U(t_j + \tau_i)$$

$$\bar{R}_{UV}(\tau_i) = \frac{1}{N} \sum_{j=1}^{N-i} U(t_j)V(t_j + \tau_i)$$

and similarly for $\bar{R}_{VV}(\tau_i)$ and $\bar{R}_{VU}(\tau_i)$. These covariance functions are computed for lags out to one-third of the record length.[3]

The results of the covariance computations are then used to compute the one-sided cosine transform of $1/2\left(\bar{R}_{UU}(\tau_i) + \bar{R}_{VV}(\tau_i)\right)$ and the one-sided sine transform of $1/2\left(\bar{R}_{UV}(\tau_i) - \bar{R}_{VU}(\tau_i)\right)$. These transformations are consistent with Eqs. (7) and (9).

Spectral averaging is then performed on the results of the cosine and sine transformations. Spectral averaging is necessary because the Fourier transform of the sample covariance function will not converge to the true power spectrum even for an infinite number of samples.[4] This computation may be viewed as obtaining an estimate of the average of the true power spectrum in a small region about a particular frequency as opposed to obtaining a point estimate of the true power spectrum at that frequency. The function that defines the small region over which the spectrum is averaged is called a spectral window.[3,4,5] The program uses the hanning spectral window[3] which performs the following simple averaging on the results of the cosine and sine transformations:

$$A(0) = 1/2\ P(0) + P(1/2NT)$$

$$A(k/2NT) = \frac{1}{4} P(k - 1/2NT) + \frac{1}{2} P(k/2NT) + \frac{1}{4} P(k + 1/2NT)$$

$$\text{for } 1 \le k \le N - 1$$

$$A(1/2T) = 1/2\ P(N - 1/2NT) + 1/2\ P(1/2T)$$

where P(k/2NT) is the result of the cosine or sine transformation at frequency k/2NT, and A(k/2NT) is the resulting spectral average at frequency k/2NT.

The program then corrects for the prewhitening initially performed on the input data. This is accomplished by merely dividing the spectral average A(k/2NT) by the quantity

$$1.36 - 1.20\cos\pi\frac{k}{N} \quad .$$

Finally, Doppler profile estimates are obtained for positive Doppler frequencies by summing the averaged and corrected sine and cosine transformations, while estimates for negative Doppler frequencies are obtained by subtracting the averaged and corrected sine transformation from the averaged and corrected cosine transformation. See Fig. 1 for a flow chart of the power spectrum program.

The power spectrum program was operated in two modes, a 1-minute mode and a 2-minute mode. The 1-minute mode processes 1,200 samples of amplitude and phase spaced at 50-msec intervals. This yields a 20-cps bandwidth with a power spectrum estimate every 1/40 cps. The 2-minute mode processes 1,200 samples of amplitude and phase spaced at 100-msec intervals. This yields a 10-cps bandwidth with a power spectrum estimate every 1/80 cps.

Figure 2 displays the results of processing a test case in which input data possessing tones at 1 and 3 cps were simulated. The test case illustrates the accuracy and resolution of the program. Examples of Doppler profiles measured on the Fort Monmouth to Palo Alto path (4100 km) and the Thule to Palo Alto path (5050 km) appear in Figs. 3 and 4.[1] Amplitude and phase measurements were obtained on both paths by transmitting a phase-stable CW tone at 7.366 Mc.[1]

Since the purpose of this memorandum is merely to explain a data processing technique used to compute channel Doppler profiles, the physical interpretation of the measured Doppler profiles is deferred to a final report to be issued under this contract.

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5. E. Parzen, "Mathematical Considerations in the Estimation of Spectra," Technometrics, Vol. 3, No. 2, pp. 167-190 (May 1960).

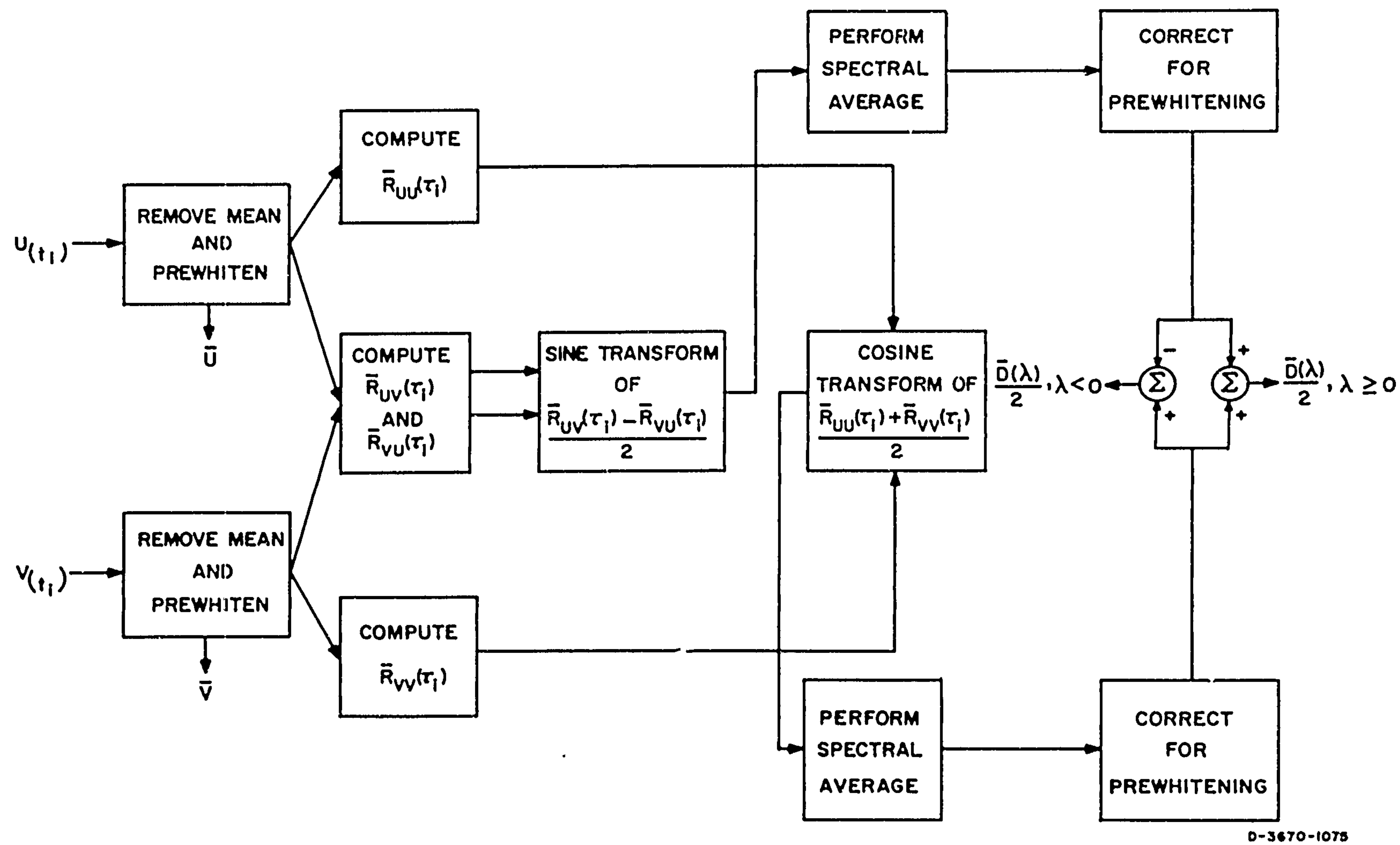

FIG. 1 DOPPLER–PROFILE POWER SPECTRUM PROGRAM FLOW CHART

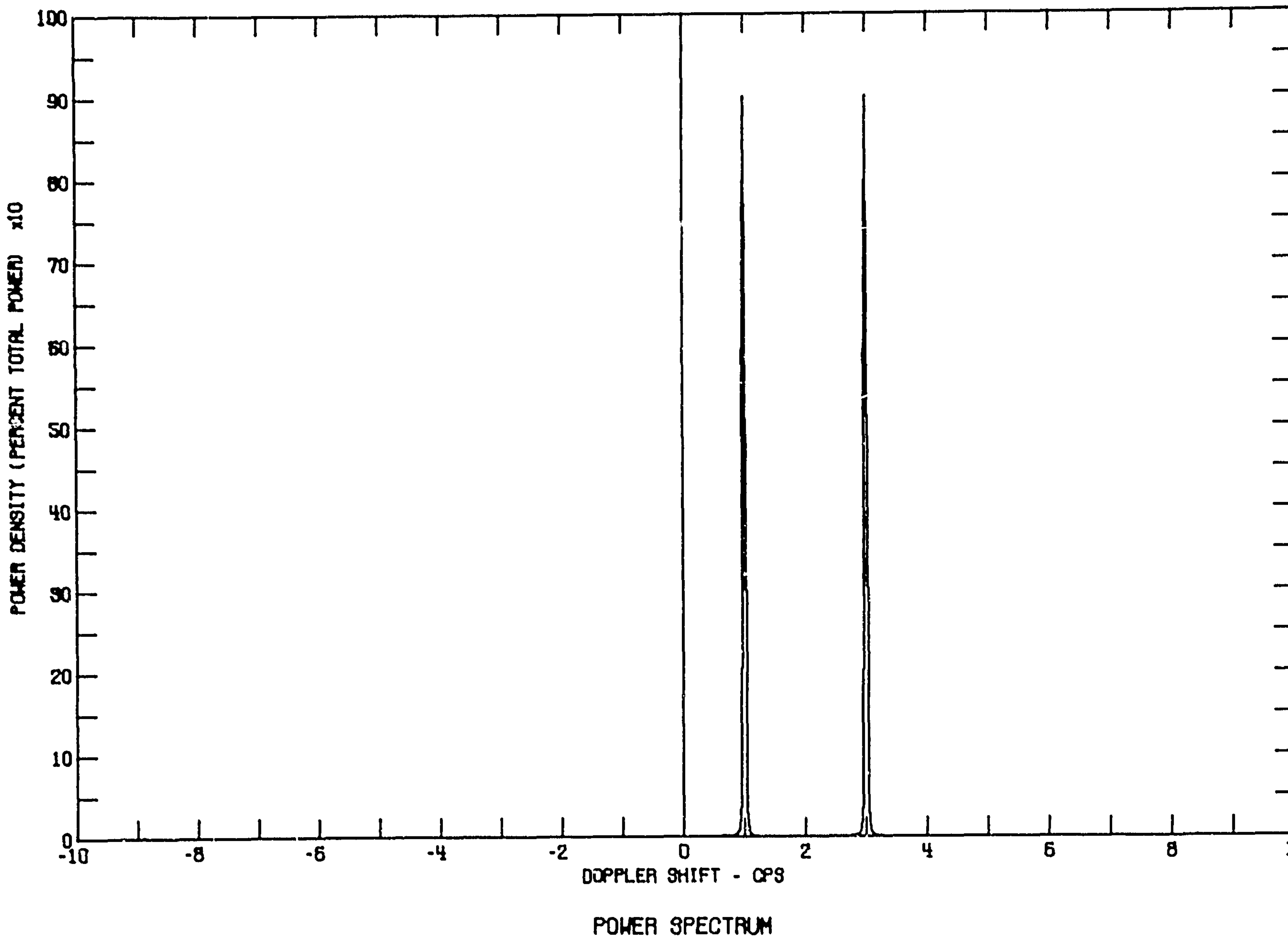

FIG. 2 DOPPLER PROFILE, 1-MINUTE MODE, TEST CASE

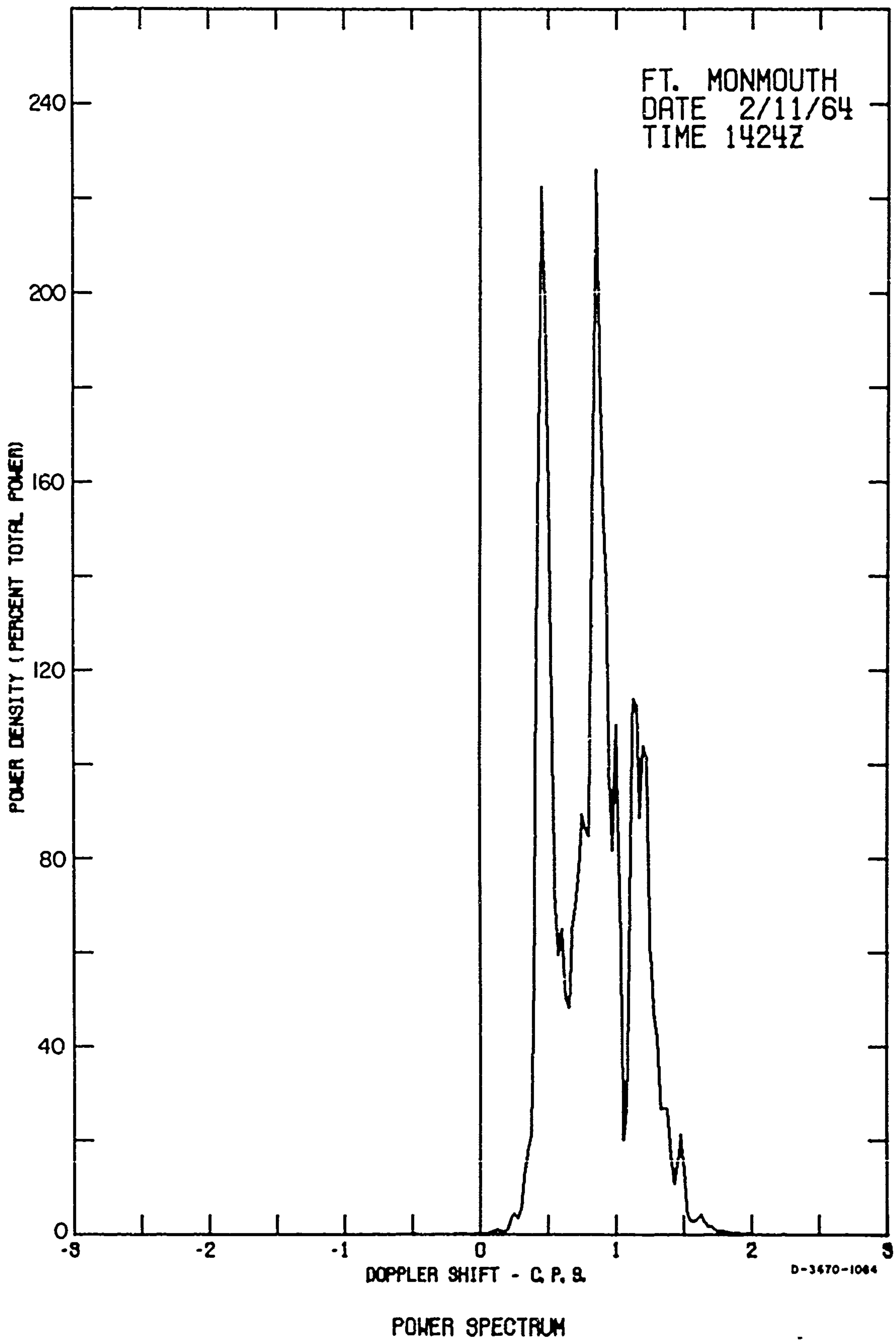

FIG. 3 DOPPLER PROFILE, 1-MINUTE MODE, FORT MONMOUTH, 11 FEBRUARY 1964, 1424Z

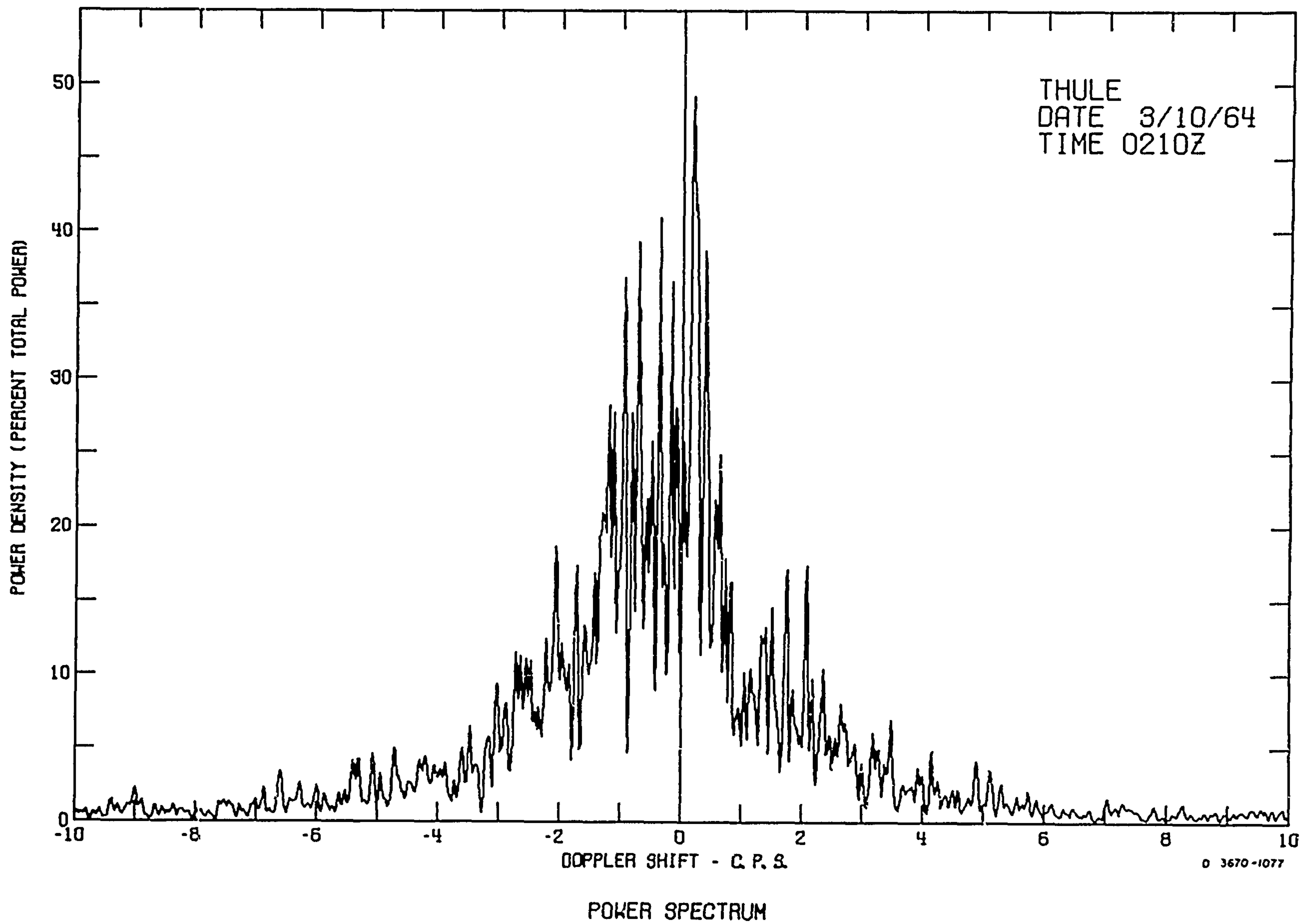

FIG. 4 DOPPLER PROFILE, 1-MINUTE MODE, THULE, 10 MARCH 1964, 0210Z

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